Model Answer of Mid Term Exam 2

2nd Mechanical Power

(I) Find the constants of the curve $y = \dfrac{1}{a\sin x + b\ln x + cx^2}$ that fit (2,101), (13,147), (20,310)

Solution

$$\frac{1}{y} = a\sin x + b\ln x + cx^2$$

$$\sum_{i=1}^{3} \frac{\sin x_i}{y_i} = a\sum_{i=1}^{3} (\sin x_i)^2 + b\sum_{i=1}^{3} e^{-x_i} \sin x_i + c\sum_{i=1}^{3} x_i^2 \sin x_i$$

$$\sum_{i=1}^{3} \frac{\ln x_i}{y_i} = a\sum_{i=1}^{3} \sin x_i \ln x_i + b\sum_{i=1}^{3} (\ln x_i)^2 + c\sum_{i=1}^{3} x_i^2 \ln x_i$$

$$\sum_{i=1}^{3} \frac{x_i^2}{y_i} = a\sum_{i=1}^{3} x_i^2 \sin x_i + b\sum_{i=1}^{3} x_i^2 \ln x_i + c\sum_{i=1}^{3} x_i^4$$

x_i	y_i	$\frac{\sin x_i}{y_i}$	$(\sin x_i)^2$	$\ln x_i \sin x_i$	$x_i^2 \sin x_i$	$\frac{\ln x_i}{y_i}$	$(\ln x_i)^2$	$x_i^2 \ln x_i$	$\frac{x_i^2}{y_i}$	x_i^4
2	101	9.0029×10^{-3}	0.8268	0.6303	3.6372	6.8628×10^{-3}	0.4805	2.7726	0.0396	16
13	147	2.8583×10^{-3}	0.1765	1.0777	71.0082	0.0174	6.5790	433.4764	1.1497	28561
20	310	2.9450×10^{-3}	0.8335	2.7349	365.1781	9.6637×10^{-3}	8.9744	1198.2929	1.2903	160000
Sum		0.0148	1.8368	4.4429	439.8235	0.0339	16.0339	1634.5419	2.4796	188577

0.0148= 1.8368 a + 4.4429 b + 439.8235 c

0.0339 =4.4429 a + 16.0339 b + 1634.5419 c

2.4796= 439.8235 a +1634.5419 b + 188577 c

a= 8.0683×10^{-3} , b=3.9224×10^{-3}, c= -3.9667×10^{-5}

$$\therefore y = \frac{1}{8.0683x10^{-3}\sin x + 3.9224x10^{-3}\ln x - 3.9667x10^{-5}x^2}$$

(II) Drive the formula to solve the following P.D.E. numerically

$$2u_t = u_{xx} \ , 0 < x < 1, \ u(0,t) = 3, \ u(1,t) = 2, \ u(x,0) = 3 - x^2, h = 0.25, k = 1$$

<u>Solution</u>

$$2\frac{u_{i,j+1} - u_{i,j}}{k} = \frac{u_{i+1,j} - 2u_{i,j} + u_{i-1,j}}{h^2}$$

$$u_{i,j+1} = \frac{k}{2h^2}(u_{i+1,j} + u_{i-1,j}) + (1 - \frac{k}{h^2})u_{i,j} \ , h = 0.25, k = 1$$

$$u_{i,j+1} = 8(u_{i+1,j} + u_{i-1,j}) - 15u_{i,j}$$

$$u_1 = u_{10} = u_{11} = 3, \ u_5 = u_6 = u_{15} = 2,$$

$$u_2 = \frac{47}{16} = 2.9375, u_3 = \frac{11}{4} = 2.75, u_4 = \frac{39}{16} = 2.4375.$$

11	12	13	14	15
10	9	8	7	6
1	2	3	4	5

$$u_9 = 8(u_3 + u_1) - 15u_2 = 8(\frac{11}{4} + 3) - 15(\frac{47}{16}) = \frac{31}{16} = 1.9375$$

$$u_8 = 8(u_4 + u_2) - 15u_3 = 8(\frac{39}{16} + \frac{47}{16}) - 15(\frac{11}{4}) = \frac{7}{4} = 1.75$$

$$u_7 = 8(u_5 + u_3) - 15u_4 = 8(2 + \frac{11}{4}) - 15(\frac{39}{16}) = \frac{23}{16} = 1.4375$$

$$u_{12} = 8(u_8 + u_{10}) - 15u_9 = 8(\frac{7}{4} + 3) - 15(\frac{31}{16}) = \frac{143}{16} = 8.9375$$

$$u_{13} = 8(u_7 + u_9) - 15u_8 = 8(\frac{23}{16} + \frac{31}{16}) - 15(\frac{7}{4}) = \frac{3}{4} = 0.75$$

$$u_{14} = 8(u_6 + u_8) - 15u_7 = 8(2 + \frac{7}{4}) - 15(\frac{23}{16}) = \frac{135}{16} = 8.4375$$